Dissolution of the Dilemma Problems by Defining the Criteria Matrix in ANP

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ABSTRACT

There are many cases with a dilemma problem where the best choice cannot be obtained based on the transitive relation. This paper presents a new method of dissolution of the dilemma problem by using ANP (Analytic Network Process). The super-matrix of ANP both with the alternative matrix of a dilemma and with defining a new criteria matrix is considered. The paper shows mathematically that the eigenvector of the super-matrix serves as a solution of the simple dilemma problem. Moreover, the descriptive interpretation of the proposed method and applications to fallacy of composition problem are performed by some examples.

Keywords: Decision analysis, ANP (Analytic Network Process), dilemma problem, eigenvector, fallacy of composition

1. Introduction

There are many cases with the dilemma problem where the best choice cannot be obtained based on the transitive relation. Especially, the activity of the enterprise of this problem is not unusual. For instance, let's think about the product planning that draws up customer's needs based on the conception of the reversal on the value chain. In such a case, the dissension and the confrontation are often seen between the producers near the customers, the development and the management department in a strategic standpoint. That is, there emerges an evaluation problem with the dilemma.

Arrow (1951) proved that irrationality is generated in the society where the evaluator is composed of multiple types of people in the general possibility theorem whenever a social decision-making is done from more than three choices. Such irrationality is often generated by the difference in the standpoint where each evaluator is left. So the decision-making technique to the evaluation problem with the dilemma is extremely important to the definition of Simon (1977) saying "To manage is to decide to make".

Because AHP (Analytic Hierarchy Process) (Saaty, 1980) is one technique of the decision-makings and requires the transitive relation, the influence of the dilemma is taken into Consitency Index (C.

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I.), and has been treated as a decision-making stress problem. In a large close-up of the evaluation problem with the dilemma in full scale, the research of reversal problem of the order in AHP is like the opportunity. Because the reversal of the order in the decision-making method becomes a fatal fault of the technique, Choo et al. (1999), Saaty (1983, 1987, 1990, 1993, 1994, 2003, 2006), Schoner et al. (1989, 1993), Belton & Gear (1983), Dyer (1990), Kinoshita et al. (1997, 1999a, 1999b, 1999c, 2002, 2008), Harker et al. (1987, 1990), and Tamura et al. (1998) have controverted each other. As a result, some methods of evading reverse are proposed. Kinoshita et al. assumed criteria weights to be dependent upon the alternatives, developed dominant AHP Linking pin AHP (Choo et al., 1999; Schoner et al., 1989, 1993) and a simple super-matrix with ANP (Saaty, 1996) all yield the same solution of dominant AHP, and dissolve the reversal of the order. Kinoshita et al. (2002) have been developing Concurrent Convergence Method and Concurrent Convergence Method of Evaluation Value to supply dominant AHP to practical use. Sugiura & Kinoshita (2005, 2007) showed the dissolution of the evaluation problem with the dilemma by using Concurrent Convergence of Evaluation Value. However, the dissolution method by ANP has not been proposed yet. This paper describes a dissolution method of the dilemma problem by using ANP and some findings to ANP by some examples. And ANP will suggest a new use method in the research in the paper.

The rest of this paper is organized as follows. In Section 2, the dissolution method of the dilemma problem is introduced. A new ANP method is proposed and two interpretations of the proposed method are described in Section 3. Some examples of application and knowledge to ANP are presented in Section 4. Some matters are discussed further in Section 5, and the conclusion of this paper is brought together in Section 6.

2. Dissolution method of the dilemma problem

In this Section, a new dissolution method of the dilemma problem is presented. First of all, the Kinoshita's method to the dilemma problem is introduced. Secondly, a new dissolution method is proposed.

2.1 Previous method

The decision-making determines a man's subsequent behavior. Therefore, the method of the problem should offer the decision-maker preferable information on the alternatives and risk information according to the selection. However, there is no decision-making method that gives significant information on the evaluation problem with circulative, for instance, "Rock＞Scissors", "Scissors＞Paper", "Rock＞Paper". Referring to the idea of Arrow (1951), Sugiura & Kinoshita (2007) classified the dilemma problems into two kinds as following; simple dilemma and dilemma by fallacy of composition, and showed the unified solution. When the alternatives are evaluated by a specific basis of selection, simple dilemma is generated. Therefore, the rock-paper-scissors becomes simple dilemma. When two or more decision-makers exist, and when decision-maker's standpoints are different, dilemma by fallacy of composition is generated.

The dilemma problem in AHP was taken up as an architectural issue by Triantaphyllou (2001) and by Sugiura et al. (2005, 2007) with comparing two alternatives for the cancellation of the order reversal problem. For instance, three types of Japanese Sinkansen; "Kodama (K)", "Hikari (H)", and "Nozomi (N)" were assumed to be mutual alternatives, and "Amenity (C1)" and "Economy (C2)" were assumed to be criteria in the illustration of simple dilemma that Sugiura & Kinoshita (2005) set. In this example, it was impossible to set priorities between three alternatives (Table 1), i.e. Kodama (0.68) > Hikari (0.32), Hikari (0.55) > Nozomi (0.45), not be transitive Kodama > Nozomi because of Nozomi (0.6) > Kodama (0.4). They took the ratios of the evaluation values, and tried the evaluation by AHP (Table 2). Base on Table 1 value data, we can get priority weights as Table 2, Kodama (0.3712) > Nozomi (0.3539) > Hikari (0.2749).

Table 1 Illustration of simple dilemma

Alternatives	Amenity C1(0.8)	Economy C2(0.2)	Value
Kodama	0.7	0.6	0.68 (a_1)
Hikari	0.3	0.4	0.32 (a_2)
Alternatives	C1(0.5)	C2(0.5)	Value
Nozomi	0.9	0.3	0.6 (b_2)
Kodama	0.1	0.7	0.4 (b_1)
Alternatives	C1(0.3)	C2(0.7)	Value
Hikari	0.2	0.7	0.55 (c_1)
Nozomi	0.8	0.3	0.45 (c_2)

Table 2 Pairwise comparison

	Kodama	Hikari	Nozomi	Eigenvalue
Kodama	1	2.1250	0.6667	0.3712 (1)
Hikari	0.4706	1	1.2222	0.2749 (3)
Nozomi	1.5000	0.8182	1	0.3539 (2)

Concurrent Convergence Method of Evaluation Value was used to verify whether the solution in AHP was appropriate together. AHP was defined in the Concurrent Convergence Method together as follows (**Appendix 1**).

$$\begin{array}{c} \\ Kodama \\ Hikari \\ Nozomi \end{array} \begin{array}{c} \begin{matrix} Kodama & Hikari & Nozomi \end{matrix} \\ \begin{bmatrix} 1 & 2.1250 & 0.6667 \\ 0.4706 & 1 & 1.2222 \\ 1.5000 & 0.8182 & 1 \end{bmatrix} \end{array} = \begin{array}{c} \\ K \\ H \\ N \end{array} \begin{array}{c} \begin{matrix} K & H & N \end{matrix} \\ \begin{bmatrix} 1 & M_2^{HK} & M_3^{NK} \\ M_1^{KH} & 1 & M_3^{NH} \\ M_1^{KN} & M_2^{HN} & 1 \end{bmatrix} \end{array},$$

$$\begin{array}{c} \\ K \\ H \\ N \end{array} \begin{array}{c} \begin{matrix} K & H & N \end{matrix} \\ \begin{bmatrix} 1 & \sqrt[3]{M_1^{HK} \cdot M_2^{HK} \cdot M_3^{HK}} & \sqrt[3]{M_1^{NK} \cdot M_2^{NK} \cdot M_3^{NK}} \\ \sqrt[3]{M_1^{KH} \cdot M_2^{KH} \cdot M_3^{KH}} & 1 & \sqrt[3]{M_1^{NH} \cdot M_2^{NH} \cdot M_3^{NH}} \\ \sqrt[3]{M_1^{KN} \cdot M_2^{KN} \cdot M_3^{KN}} & \sqrt[3]{M_1^{NH} \cdot M_2^{NH} \cdot M_3^{NH}} & 1 \end{bmatrix} \end{array} = \begin{bmatrix} 1 & 1.3505 & 1.0490 \\ 0.7405 & 1 & 0.7768 \\ 0.9533 & 1.287 & 1 \end{bmatrix}$$

All ratios of the evaluation of final alternatives became equivalent, and the priority (0.3712, 0.2749, 0.3539) was obtained as the normalized eigenvector of *K, H, N*. This result showed that the priority level was uniquely decided to such the problem as the illustration of simple dilemma.

2.2 Proposed method

AHP needs the information about all alternatives or ratios of the evaluation items (criteria) usually. However, the pairwise comparison might be difficult to evaluate it. For example, let's consider the dilemma or circulative matrix in Table 1 again.

$$\boldsymbol{U}_{\square} = \begin{array}{c} \\ K \\ H \\ N \end{array} \begin{array}{c} \begin{matrix} T_1 & T_2 & T_3 \end{matrix} \\ \begin{bmatrix} a_1 & b_1 & \square \\ a_2 & \square & c_1 \\ \square & b_2 & c_2 \end{bmatrix} \end{array} \tag{1}$$

Let's consider each element of a couple of pairwise comparison matrices shown by, A, B and C.

$$\boldsymbol{U}_{AHP} = \begin{array}{c} \\ K \\ H \\ N \end{array} \begin{array}{c} \begin{matrix} K & H & N \end{matrix} \\ \begin{bmatrix} a_1/a_1 & a_1/a_2 & b_1/b_2 \\ a_2/a_1 & a_2/a_2 & c_1/c_2 \\ b_2/b_1 & c_2/c_1 & c_2/c_2 \end{bmatrix} \end{array} = \begin{array}{c} \\ K \\ H \\ N \end{array} \begin{array}{c} \begin{matrix} K & H & N \end{matrix} \\ \begin{bmatrix} 1 & A & B \\ 1/A & 1 & C \\ 1/B & 1/C & 1 \end{bmatrix} \end{array} \tag{2}$$

Where, $A = a_1/a_2$, $B = b_1/b_2$ and $C = c_1/c_2$.

We think about ANP that sets these two matrixes by assuming the matrix that interpolates the loss of the evaluation value to be $\boldsymbol{W}_{\square}$. So, the element of no evaluation is assumed to be zero, then it is rewritten that $\boldsymbol{U}_{\square}$ is $\boldsymbol{U}$, and that $\boldsymbol{W}_{\square}$ is $\boldsymbol{W}$. The evaluated element is assumed to be one, ANP will be made from the criteria matrix. The super-matrix of ANP is made from the alternatives matrix $\boldsymbol{U}$ by missing value to be zero and the criteria matrix $\boldsymbol{W}$ by assuming evaluation value to be one.

$$\boldsymbol{W}=\begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix} \tag{3}$$

The definition of this criteria matrix $\boldsymbol{W}$ is insufficient. The eigenvector of $\boldsymbol{UW}$ is, (0.3620, 0.2778, 0.3602), the order is maintained, and the error margin with each eigenvector is within ± 0.01. Therefore, we may obtain the same solution as equation (2) by ANP, if suitable criteria matrix $\boldsymbol{W}$ is defined. Then, let's think about the eigenvector of ANP based on the criteria matrix $\boldsymbol{W}$ by taking the matrix inverse of $\boldsymbol{U}$.

$$\boldsymbol{W}=\begin{bmatrix} \frac{b_2c_1}{a_1b_2c_1+a_2b_1c_2} & \frac{b_1c_2}{a_1b_2c_1+a_2b_1c_2} & 0 \\ \frac{a_2c_2}{a_1b_2c_1+a_2b_1c_2} & 0 & \frac{a_1c_1}{a_1b_2c_1+a_2b_1c_2} \\ 0 & \frac{a_1b_2}{a_1b_2c_1+a_2b_1c_2} & \frac{a_2b_1}{a_1b_2c_1+a_2b_1c_2} \end{bmatrix} \tag{4}$$

The eigenvector to the maximum eigenvalue k of this ANP matrix is assumed to be vectors $\boldsymbol{x}$ and $\boldsymbol{z}$.

$$\begin{bmatrix} \boldsymbol{0} & \boldsymbol{W} \\ \boldsymbol{U} & \boldsymbol{0} \end{bmatrix}\begin{bmatrix} \boldsymbol{x} \\ \boldsymbol{z} \end{bmatrix}=k\begin{bmatrix} \boldsymbol{x} \\ \boldsymbol{z} \end{bmatrix} \tag{5}$$

The overall judgement of the alternatives is shown by eigenvector $\boldsymbol{z}$ to the maximum eigenvalue. Let's compare this eigenvector $\boldsymbol{z}$ and the eigenvector $\boldsymbol{y}$ at the maximum eigenvalue α in equation (2). By assuming to be $\boldsymbol{y}(y_1, y_2, y_3)$ and $y_3=1$, each element of the eigenvector is obtained; $y_1=\frac{B\{AC(1-\alpha)-B\}}{B(1-\alpha)-AC}$ and $y_2=\frac{C\{B(1-\alpha)-AC\}}{AC(1-\alpha)-B}$ (See **Appendix 2**).

On the other hand, because the eigenvector $\boldsymbol{z}(z_1, z_2, z_3)$ is rewritten by $1-\frac{1}{1-k^2}=\alpha-1$ as $z_3=1$, following equations are obtained (See **Appendix 3**)

$$z_1=\frac{B\{(B+AC)(1-k^2)-AC\}}{(B+AC)(1-k^2)-B}=\frac{B\{B-(1-\alpha)AC\}}{AC-(1-\alpha)B}=\frac{B\{(1-\alpha)AC-B\}}{(1-\alpha)B-AC}=y_1$$

$$z_2=\frac{C\{(B+AC)(1-k^2)-B\}}{(B+AC)(1-k^2)-AC}=\frac{C\{AC-(1-\alpha)B\}}{B-(1-\alpha)AC}=\frac{C\{(1-\alpha)B-AC\}}{(1-\alpha)AC-B}=y_2.$$

Thus, in spite of α and k, the eigenvector of $\boldsymbol{UW}$ is the same as the eigenvector of $\boldsymbol{U}_{AHP}$. Therefore, such a dilemma problem can be solved by this proposed method by calculating the eigenvector in the descriptive ANP to the maximum eigenvalue.

3. Descriptive interpretation of the proposal method

The missing value is assumed to be zero in the preceding section, and the evaluation matrix is defined. In this section, the process of the decision-making is described by considering the evaluation of dissatisfaction, and the criteria matrix is interpreted like the missing value.

It is mathematically shown that equation (5) has the same eigenvector as equation (2). At this stage, the meaning of $\boldsymbol{W}$ will be defined by a Figure interpretation in Section 3.1–3.3. Let's assume the alternatives matrix with the missing value to be $\boldsymbol{U}_{\square}$, and assume the criteria matrix with the dissatisfaction for this evaluation value to be $\boldsymbol{W}_{\square}$. The alternatives, "Kodama", "Hikari", and "Nozomi", are assumed to be A_i ($i=1, 2, 3$), the situation in which virtual different evaluator T_1, T_2, and T_3 of three people evaluate alternatives is considered.

$$\boldsymbol{U}_{\square}=\begin{array}{r} \\ \textit{Kodama: } A_1 \\ \textit{Hikari: } A_2 \\ \textit{Nozomi: } A_3 \end{array}\begin{array}{c} \begin{array}{ccc} T_1 & T_2 & T_3 \end{array} \\ \begin{bmatrix} a_1 & b_1 & \square \\ a_2 & \square & c_1 \\ \square & b_2 & c_2 \end{bmatrix} \end{array} \tag{6}$$

For instance, the evaluation value given to alternative "Kodama" is either a_1 or b_1, and the evaluation value in which other alternatives are similarly shown to alternatives matrix $\boldsymbol{U}_{\square}$ is given respectively. However, it is impossible to set priorities only by the alternatives matrix $\boldsymbol{U}_{\square}$. Here, let's define the new matrix $\boldsymbol{W}_{\square}$ as the criteria matrix $\boldsymbol{W}$ based on the inverse matrix of $\boldsymbol{U}$ was introduced in this paper. We assumed the one that the matrix $\boldsymbol{W}_{\square}$ on with the evaluation value. It is necessary to make the priorities that alternatives tell the evaluators their dissatisfaction accurately. Therefore, to measure dissatisfaction accurately, the combination a_i, b_j, c_k from which the evaluation value is given to all alternatives has to be considered. The evaluations of dissatisfaction to the evaluation values of the alternatives are considered in this Section.

3.1 The evaluation of dissatisfaction to the evaluation value of the "Kodama"

(1) The assessment is the case with the evaluator T_1 (Fig. 1).

The conversion coefficient is assumed to be $1/R$, and the evaluation of the dissatisfaction degree is shown by the multiplication of the evaluation value of the other two alternatives by the evaluation value of "Kodama". The element of the row of $\boldsymbol{W}_{\square}$ corresponds to the element of the line of $\boldsymbol{U}_{\square}$ in ANP. Then in the left Figure, the dissatisfaction degree becomes $b_2 c_1/R$ times for evaluation value a_1 of "Kodama". Therefore, two alternatives need $a_1 b_2 c_1/R$ from the evaluator T_1 so that they are satisfied. Moreover, the dissatisfaction degree in the right Figure is $a_2 c_2/R$ times against the evaluation value b_1 of "Kodama", the need value to evaluator T_1 is $a_2 b_1 c_2/R$. Therefore, the satisfaction rating of "Kodama" that evaluator T_1 has to judge becomes $a_1 b_2 c_1/R+a_2 b_1 c_2/R$.

(2) The assessment is the case with the evaluator T_2 (Fig. 2).

As for the evaluator, two alternatives are not appreciable at the same time, the case of no evaluation is generated in "Hikari". The dissatisfaction degree becomes $b_1 c_2 /R$ times for evaluation value a_1 of "Kodama". Therefore, two alternatives need $a_1 b_1 c_2 /R$ from the evaluator T_2 so that they are satisfied.

(3) The assessment is the case with the evaluator T_3 (Fig. 3).

In this case, the evaluation value is given to "Kodama" excluding the evaluator T_1. The dissatisfaction degree becomes $a_1 c_1 /R$ times for evaluation value b_1 of "Kodama", and $a_1 c_1 c_1 /R$ will be needed from the evaluator T_3. Here, the following matrix $\boldsymbol{W}_{\square}$ shows the dissatisfaction degree to the evaluation value of "Kodama".

$$\boldsymbol{W}_{\square}=\begin{bmatrix} b_2 c_1 /R & b_1 c_2 /R & \square \\ a_2 c_2 /R & \square & a_1 c_1 /R \\ \square & \square & \square \end{bmatrix} \tag{7}$$

3.2 The evaluation of dissatisfaction to the evaluation value of "Hikari"

(1) The assessment is the case with the evaluator T_1 (Fig. 4).

The value of a_2 and c_1 to "Hikari" is given from the evaluator T_1 and the evaluator T_3 respectively. The dissatisfaction degree must be $b_2 c_1 /R$ in the ratio to "Hikari" of the evaluator T_1 to the value a_2. Moreover, the need value to the evaluator T_1 is $a_2 b_2 c_1 /R$. However, because the element of $\boldsymbol{U}_{\square}$ of the second row of two lines is a blank, the dissatisfaction cannot be shown in the ratio to the evaluation value of "Hikari" by this combination.

(2) The assessment is the case with the evaluator T_2 (Fig. 5).

There are two combinations in this case. The standard of dissatisfaction becomes c_1 according to the evaluation of T_2 to "Nozomi" in the left Figure. Then, the dissatisfaction degree becomes $a_1 b_2 /R$ times for the evaluation value c_1 of "Hikari". Therefore, two alternatives need $a_1 b_2 c_1 /R$ from the evaluator T_2 so that they are satisfied.

The dissatisfaction degree in the right Figure is $b_1 c_2 /R$ times to the evaluation value a_2 of "Hikari", the need value to evaluator T_2 is $a_2 b_1 c_2 /R$. Therefore, the satisfaction rating of "Hikari" that evaluator T_2 has to judge become $a_1 b_2 c_1 /R + a_2 b_1 c_2 /R$.

(3) The assessment is the case with the evaluator T_3 (Fig. 6).

The value of a_2 and c_1 to "Hikari" is given from the evaluator T_1 and the evaluator T_3 respectively. The dissatisfaction degree must be $a_2 b_1 /R$ in the ratio to "Hikari" of the evaluator T_3 to the value. c_1 Moreover, the need value to the evaluator T_3 is $a_2 b_1 c_1 /R$. However, because the element of $\boldsymbol{U}_{\square}$ of the second row of two lines is a blank, the dissatisfaction cannot be shown in the ratio as well as the paragraph (1).

The dissatisfaction degree to the evaluation value of "Hikari" is shown as follows.

$$W_{\square}=\begin{bmatrix} \square & b_1 c_2/R & \square \\ \square & \square & \square \\ \square & a_1 b_2/R & \square \end{bmatrix} \tag{8}$$

3.3 The evaluation of dissatisfaction to the evaluation value of the "Nozomi"

(1) The assessment is the case with the evaluator T_1 (Fig. 7).

The value of b_2 and c_2 to "Nozomi" is given from the evaluator T_2 and the evaluator T_3 respectively. The standard of dissatisfaction becomes b_2 according to the evaluation of T_2 to "Nozomi". Then, the dissatisfaction degree becomes $a_2 c_2/R$ times for evaluation value b_2 of "Nozomi". Two alternatives need $a_2 b_2 c_2/R$ from the evaluator T_1 so that they are satisfied.

(2) The assessment is the case with the evaluator T_2 (Fig. 8).

The value of b_2 and c_2 to "Nozomi" is given from the evaluator T_2 and the evaluator T_3 respectively. The standard of dissatisfaction becomes c_2 according to the evaluation of T_2 to "Nozomi". Then, the dissatisfaction degree becomes $a_1 b_2/R$ times for evaluation value b_2 of "Nozomi". Two alternatives need the value $a_1 b_2 c_2/R$ from the evaluator T_2 so that they are satisfied.

(3) The assessment is the case with the evaluator T_3 (Fig. 9).

There are two combinations in this case. The standard of dissatisfaction is b_2 according to the evaluation of T_3 to "Hikari" in the left figure. Then, the dissatisfaction degree becomes $a_1 c_1/R$ times for evaluation value b_2 of "Nozomi". Two alternatives need a$_1$ $b_2 c_1/R$ from the evaluator T_3 so that they are satisfied. The dissatisfaction degree in the right figure is $a_2 b_1/R$ times to the evaluation value c_2 of "Nozomi", the need value from the evaluator T_3 is $a_2 b_1 c_2/R$. Therefore, the satisfaction rating of "Nozomi" that evaluator T_3 has to judge become $a_1 b_2 c_1/R+a_2 b_1 c_2/R$.

The dissatisfaction degree to the evaluation value of "Nozomi" is shown as follows.

$$W_{\square}=\begin{bmatrix} \square & \square & \square \\ a_2 c_2/R & \square & \square \\ \square & a_1 b_2/R & a_2 b_1/R \end{bmatrix} \tag{9}$$

Finally, the matrix $W_{\square}$ left three undefined parts is obtained from equations of (7)-(9). Therefore, the need value of the satisfaction rating required is the product of the matrix $U_{\square}$ with the missing value, and it is shown as follows.

$$\begin{aligned} U_{\square}W_{\square} &= \begin{bmatrix} a_1 & b_1 & \square \\ a_2 & \square & c_1 \\ \square & b_2 & c_2 \end{bmatrix} \cdot \begin{bmatrix} b_2 c_1/R & b_1 c_2/R & \square \\ a_2 c_2/R & \square & a_1 c_1/R \\ \square & a_1 b_2/R & a_2 b_1/R \end{bmatrix} \\ &= \begin{bmatrix} (a_1 b_2 c_1+a_2 b_1 c_2)/R & a_1 b_1 c_2/R & a_1 b_1 c_1/R \\ a_2 b_2 c_1/R & (a_1 b_2 c_1+a_2 b_1 c_2)/R & a_2 b_1 c_1/R \\ a_2 b_2 c_2/R & a_1 b_2 c_2/R & (a_1 b_2 c_1+a_2 b_1 c_2)/R \end{bmatrix} \end{aligned} \tag{10}$$

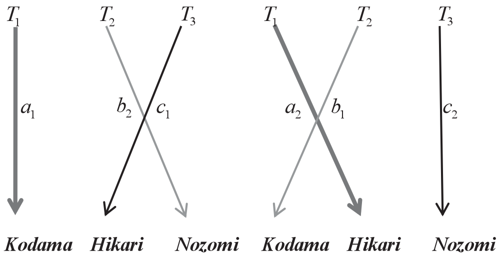

Fig. 1 Case where evaluator T_1 judges evaluation of dissatisfaction rating

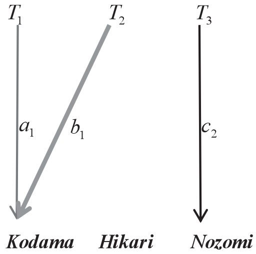

Fig. 2 Case where evaluator T_2 judges evaluation

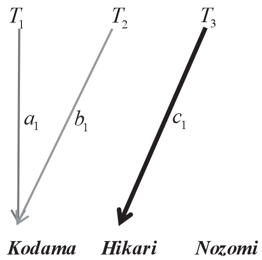

Fig. 3 Case where evaluator T_3 judges evaluation

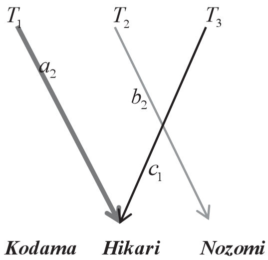

Fig. 4 Case where evaluator T_1 Judges evaluation

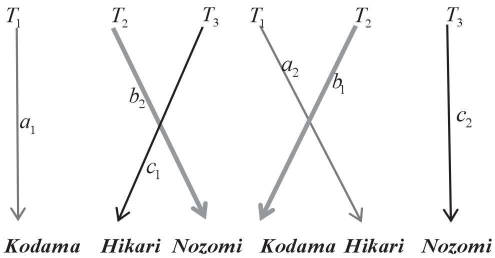

Fig. 5 Case where evaluator T_2 judges evaluation

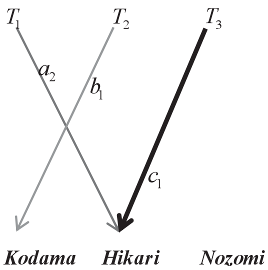

Fig. 6 Case where evaluator T_3 judges evaluation

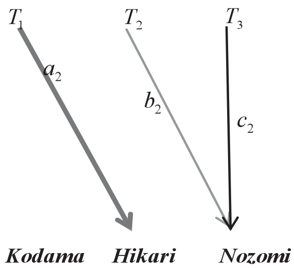

Fig. 7 Case where evaluator T_1 judges evaluation

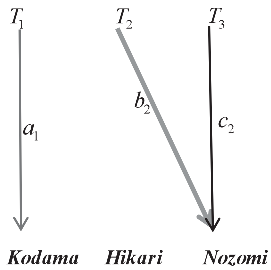

Fig. 8 Case where evaluator T_2 Judges evaluation

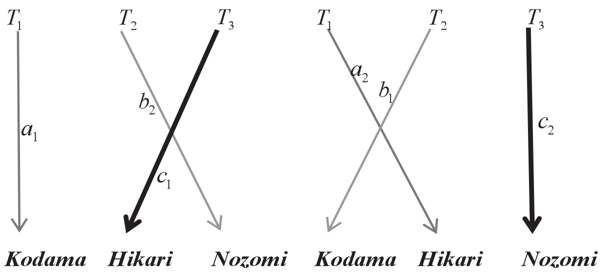

Fig. 9 Case where evaluator T_3 judges evaluation

The equation (10) holds even if the missing value is assumed to be zero, and the need for the evaluators in equations of (1) and (3) of the paragraph 3.2 can be achieved. Therefore, the equation of $\boldsymbol{U}_{\square}\boldsymbol{W}_{\square}$ can be concluded to be $\boldsymbol{UW}$ by assuming the blanks to be zero from the meaning of the missing value "There is no evaluation".

By the way, the elements other than the missing value in the matrix $\boldsymbol{W}_{\square}$ are same of the inverse matrix of $\boldsymbol{U}$, then $\boldsymbol{W}$ shows the ratio of dissatisfaction to the element of $\boldsymbol{U}$ from the figure interpretation. Because the position of zero does not change when the position of zero is symmetry in the matrix of $\boldsymbol{U}$, $\boldsymbol{W}$ is obtained by taking the inverse matrix of $\boldsymbol{U}$ and by replacing the missing value position with zero, and shown by the equation (4) when assuming $R = a_1 b_2 c_1 + a_2 b_1 c_2$.

4. Examples of application and knowledge to ANP

Up to now, the simple dilemma problem is taken up. However, the dilemma problem that Triantaphyllou pointed out, and a dilemma problem by fallacy of composition, have been left. In this Section, some examples are illustrated to show the utility of the propose method.

4.1 Dilemma problem of Triantaphyllou

Table 3 and Table 4 are cases where the order reversal that Tiantaphyllou (2001) pointed out is caused in AHP. Such a dilemma problem can be occurred easily when there are a lot of criteria and alternatives. So, this proposed method will be applied to Example 1 and Example 2 of Triantaphyllou.

The priority level of alternative A1, A2, and A3 is A3 > A2 > A1 in Example 1, and A2 > A3 > A1 in Example 2. However, the eigenvector by the pair wise comparison of three alternatives is (0.3505, 0.3319, 0.3176) in Example 1, and (0.9213, 0.3696, 0.3391) in Example 2.

When the proposed method is applied to these examples, $\boldsymbol{UW}_1$ in Example 2 and $\boldsymbol{UW}_2$ in Example 2 are as follows respectively.

$$\boldsymbol{UW}_1 = \begin{bmatrix} 1 & 0.5232 & 0.5540 \\ 0.4777 & 1 & 0.5202 \\ 0.4512 & 0.4804 & 1 \end{bmatrix} \text{and } \boldsymbol{UW}_2 = \begin{bmatrix} 1 & 0.4090 & 0.4056 \\ 0.5992 & 1 & 0.5656 \\ 0.6042 & 0.4332 & 1 \end{bmatrix}$$

Each eigenvector becomes (0.3499, 0.3325, 0.3176), i.e. A1 > A2 > A3 and (0.2913, 0.3396, 0.3391), i.e. A2 > A3 > A1 and is almost the same as the value that Triantaphyllou (2001) calculated. Thus, two examples of point out that a dilemma problem may be generated when two or more alternatives are evaluated simultaneously or the differences of the evaluation values are a little. The method of Triantaphyllou is the same as the method of Kinoshita et al. (2005), who showed in equation (2) in the point to use AHP. Unfortunately, he only examined the reversal of the order, and did not specify the method of the dissolution with dilemma evaluation. It can be said that this proposed method is more

Table 3. Example 1 of Triantaphyllou

	C1(2/7)	C2(2/7)	C3(3/7)	Evaluation
A1	9/19	2/12	2/7	0.3054
A2	5/19	1/12	4/7	0.3439
A3	5/19	9/12	1/7	0.3507

A3 > A2 > A1

	C1(2/7)	C2(2/7)	C3(3/7)	Evaluation
A1	9/14	2/3	2/6	0.5170
A2	5/14	1/3	4/6	0.4830

	C1(2/7)	C2(2/7)	C3(3/7)	Evaluation
A2	5/10	1/10	4/5	0.5143
A3	5/10	9/10	1/5	0.4857

	C1(2/7)	C2(2/7)	C3(3/7)	Evaluation
A1	9/14	2/11	2/3	0.5213
A3	5/14	9/11	1/3	0.4787

A1 > A2 > A3

Table 4. Example 1 of Triantaphyllou

	C1(4/22)	C2(9/22)	C3(9/22)	Evaluation	Normalize
A1	9/9	5/8	2/8	0.5398	0.2844
A2	1/9	8/8	5/8	0.6850	0.3609
A3	8/9	2/8	8/8	0.6730	0.3546

A2 > A3 > A1

	C1(4/22)	C2(9/22)	C3(9/22)	Evaluation	Normalize
A2	1/8	8/8	5/8	0.6875	0.4979
A3	8/8	2/8	8/8	0.6932	0.5021

	C1(4/22)	C2(9/22)	C3(9/22)	Evaluation	Normalize
A1	9/9	5/8	2/5	0.6011	0.4176
A2	1/9	8/8	5/5	0.8384	0.5824

	C1(4/22)	C2(9/22)	C3(9/22)	Evaluation	Normalize
A1	9/9	5/8	2/8	0.6932	0.4856
A3	1/9	8/8	5/8	0.7343	0.5144

A3 > A2 > A1

descriptive and excellent than the method of Triantaphyllou because the dissolution of the dilemma problem is described by ANP that clarifies the interaction of alternatives and criteria.

4.2 Discussion of the dilemma problem with fallacy of composition

A fallacy of composition arises when the whole is inferred from the fact that each choice is appropriate of some part of the whole. In recent years, the life cycle of the commodity is short, and the development risk has risen to, since the value-chain reverses and the product development is indispensable based on customers' needs. Therefore, the decision-making problem is important to solve in the enterprise.

For instance, let's assume the case of enterprise where the evaluation of a certain product planning separates mutually in the farm sector, and the development sector and the production like the satisfaction rating shown in Table 5. Let's apply the proposed method to this illustration.

The farm sector evaluates the order of Commodity 1 (83) > Commodity 2 (77) > Commodity 3 (64) and the development sector evaluates the order of Commodity 3 (85) > Commodity 1 (72) > Commodity 2 (56). On the other hand, the evaluation of the production sector evaluates the order of Commodity 3(70) > Commodity 2(85) > Commodity 1 (65). Therefore a fallacy of composition

Table 5 Dilemma problem with fallacy of composition

Planning product	Enterprise sector			Total value
	Farm	Development	Production	
Commodity 1	83	72	65	220
Commodity 2	77	56	85	218
Commodity 3	64	85	70	219

arises. However, it is impossible to set priorities in total about the evaluation point because there is a little difference in the total value of each section. Then, the evaluation matrix $\boldsymbol{UW}$ is made based on Commodity 1 in the first row, Commodity 2 in the second row and Commodity 3 in the third row.

$$\boldsymbol{UW}=\begin{bmatrix} 1 & 1.2857 & 0.9286 \\ 0.9277 & 1 & 1.2143 \\ 0.7711 & 1.5186 & 1 \end{bmatrix} \tag{9}$$

Moreover, the matrix is made by crossing the element of each line of $\boldsymbol{UW}$.

$$\left[\frac{A^2B^2C}{(B+AC)^2} \ \frac{BC^2}{(B+AC)^2} \ \frac{A}{(B+AC)^2}\right]^T=[1.1939 \quad 1.1265 \quad 1.1704]^T$$

By neglecting the denominator of the element, $A=1.1704$, $B=0.8769$ and $C=1.1334$ are obtained. Finally, the same shape of equation (1) is obtained. Then, we consider the eigenvector of the super-matrix based on the criteria matrix $\boldsymbol{W}$ by taking the inverse matrix of $\boldsymbol{U}$.

$$\begin{bmatrix} \boldsymbol{0} & \boldsymbol{W} \\ \boldsymbol{U} & \boldsymbol{0} \end{bmatrix}=\begin{bmatrix} 0 & 0 & 0 & 0.5144 & 0.3980 & 0 \\ 0 & 0 & 0 & 0.4538 & 0 & 0.6020 \\ 0 & 0 & 0 & 0 & 0.5312 & 0.3980 \\ 1.1704 & 0.8769 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1.1334 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \end{bmatrix}$$

This eigenvector of the Commodity is (0.3362, 0.3298, 0.3340), and the priority level becomes Commodity 1 > Commodity 3 > Commodity 2.

By the way, when the alternative's matrix $\boldsymbol{U}$ is seen, the evaluation of the farm enterprise is Commodity 1 (1.1704) > Commodity 2 (1), the evaluation of the development section is Commodity 1 (0.8769) < Commodity 3 (1), and the evaluation of the production department is Commodity 2 (1.1334) > Commodity 3 (1). Considering only the farm enterprise and the development section, product planning and evaluation of Commodity 3, the priority level becomes Commodity 1 (1.1704) > Commodity 3 (1.3347) > Commodity 2 (1) because the evaluation of Commodity 3 is (1.1704/0.8769

= 1.334). However, because the evaluation of the production department is Commodity 2 (1.1334) > Commodity 3 (1), the dilemma of the evaluation is caused in the priority level shown in Table 1. It is shown that the fallacy of composition can be derived from simple dilemma, and that the dilemma problem accompanied fallacy of composition can be dissolved by this proposed method.

5. Discussion and Implications

Though this paper is a description of the proposed method and an application of dilemma problems by using ANP, some matters are discussed and implicated further.

(1) Meaning of the criteria matrix $\boldsymbol{W}$

The kernel of the present paper is to have found the criteria matrix $\boldsymbol{W}$ based on the matrix inverse to the alternatives matrix $\boldsymbol{U}$ where a dilemma problem is occurred. In other words, it is that is because the dilemma problem can be dissolved by using ANP that consists of the alternatives matrix $\boldsymbol{U}$ and the criteria matrix $\boldsymbol{W}$.

By the way, “All alternatives are dissatisfied with the evaluation” is specified in Section 2, because the basis of $\boldsymbol{W}$ is the matrix inverse of $\boldsymbol{U}$. However, it is uncertain in Section 3 whether the assumption is consistently reflected in the criteria matrix $\boldsymbol{W}$. For instance, the dissatisfaction degree is defined by the product of the evaluation values of other alternatives to the element of the evaluation value, but it is not specified whether it has exhibited dissatisfaction. Let’s think about the evaluator’s judgment of the need, because the dissatisfaction is appearance of the need. It is assumed that the evaluator T_1, T_2, and T_3 give the evaluation value of z_1, z_2, and z_3 to the need of alternatives respectively and assumed $z_1+z_2+z_3=1$. For example, the evaluator T_1 gives the evaluation value z_1 of one or less to need value $(a_1 b_2 c_1+a_2 b_1 c_2)/R$ of “Kodama”. Moreover, the evaluator T_2 gives z_2 to need value $a_1 b_1 c_2 /R$ and z_3 to need value. $a_1 b_1 c_1 /R$ These evaluation values of z_1, z_2, and z_3 have reduced the need value.

That is, an excessive need has been generated because all alternatives are dissatisfied with the evaluation value. The evaluator should attempt to make of the evaluation an aptitude by reducing the dissatisfied value in each alternative. Therefore, the sum of the evaluation value with the need value of “Kodama” reaches the proper evaluation value. It is induced that the evaluation values of z_1, z_2, and z_3 are eigenvectors of $\boldsymbol{UW}$ because the evaluator T_1, T_2, and T_3 similarly do a proper evaluation to the need values of “Hikari” and “Nozomi”. Thus, the criteria matrix $\boldsymbol{W}$, shown the standard of dissatisfaction in the ratio on the assumption of “All alternatives are dissatisfied”, plays an important role of composing $\boldsymbol{UW}$ and of obtaining a proper evaluation, based on the concept, the alternative matrix $\boldsymbol{U}$ and the criteria matrix W are independence and feed back.

(2) Normalization of ANP

Though the eigenvector of ANP that alternative matrix $\boldsymbol{U}$ in Example 2 is normalized is not changed, the eigenvector of the super-procession that criteria matrix $\boldsymbol{W}$ is normalized becomes (0.2818, 3799, 3382) and doesn't reach the same value. This reason is thought as follows. ANP that consists of the alternative matrix $\boldsymbol{U}$ and criteria matrix $\boldsymbol{W}$ is shown as follows using A, B and C.

$$\begin{bmatrix} Aa_2 & Bb_2 & 0 \\ a_2 & 0 & Cc_2 \\ 0 & b_2 & c_2 \end{bmatrix} \cdot \begin{bmatrix} \frac{C}{a_2(AC+B)} & \frac{B}{a_2(AC+B)} & 0 \\ \frac{1}{b_2(AC+B)} & 0 & \frac{AC}{b_2(AC+B)} \\ 0 & \frac{A}{c_2(AC+B)} & \frac{B}{c_2(AC+B)} \end{bmatrix} \begin{bmatrix} 1 & \frac{AB}{B+AC} & \frac{ABC}{B+AC} \\ \frac{C}{B+AC} & 1 & \frac{BC}{B+AC} \\ \frac{1}{B+AC} & \frac{A}{B+AC} & 1 \end{bmatrix} \tag{10}$$

Though the alternative matrix $\boldsymbol{U}$ is not normalized, this equation (10) shows that the values of A, B and C do not change. In a word, the eigenvector of $\boldsymbol{UW}$ is not changed even though A, B and C take what positive number. However, when the criteria matrix $\boldsymbol{W}$ is normalized, neither a_2, b_2 nor c_2 are independent respectively, and the element of $\boldsymbol{UW}$ is also different from equation (10). That is, the criteria matrix $\boldsymbol{W}$ cannot be normalized. In AHP and ANP, it has been said to be natural that the criteria matrix $\boldsymbol{W}$ and the alternative matrix $\boldsymbol{U}$ are normalized. Moreover, this result suggests having not to normalize the criteria matrix easily though normalization is required in the super-matrix of Saaty.

(3) A characteristic of this proposed method of fallacy of composition problem.

A dilemma problem has been handled as a whole by Section 4.2, though there is consistency in the evaluation in each section. This eigenvector of the Commodity is (0.3362, 0.3298, 0.334, 0), and the priority level becomes Commodity 1 ＞ Commodity 3 ＞ Commodity 2. However, it is not easy to obtain agreement actually because the difference of the element of the eigenvector is small.

When A, B, and C obtained from equation (10) are 1.1704, 0.8769, and 1.1334 respectively, the eigenvector is obtained as (0.3362, 0.3298, 0.3340). The priority is calculated by using the numerical value in Table 5 for a_2, b_2, and c_2 to enlarge the difference of the element of the eigenvector (Case 1), i.e. 70*1.1704＝90, 85*0.8769＝75, 70*1.1334＝79. Next, the priority will be calculated by using a_1, b_1, and c_1 (Case 2), i.e. 70*1.1704＝90, 85*0.8769＝75, 70*1.1334＝79. In Case1 to which the difference between the low rank and most significant is expanded, the difference of the element of the eigenvector is greater than that of the previous one, i.e. (0.3448, 0.3164, 0.3380). In Case 2, the difference between the low rank and the neutral position is expanded, and the reversal of the order is caused, i.e. Commodity 1 (0.3379) ＜ Commodity 3 (0.3396). Therefore, the correction like Case 1 is preferable to maintain the order. Therefore, the review of the evaluation point has to be attempted, the correction like Case1 will be preferable to maintain the order. However, it is possible to set priorities easily by using the sum of each evaluation point in Case 1. These results are shown in Table 6.

By the way, the eigenvector $\boldsymbol{z}(z_1, z_2, z_3)$ of $\boldsymbol{UW}$ in the Fallacy of the composition problem will be examined. The product of B and C can be shown as follows by the eigenvector $\boldsymbol{w}(w_1, w_2, w_3)$ obtained

Table 6 Review of evaluation value

	Case1						Case2					
	Farm	Develop	Pro	Priority	Sum	Ratio	Farm	Develop	Pro	Priority	Sum	Ratio
Com1	90	75	65	0.3448	230	0.3480	83	72	65	0.3379	220	0.3369
Com2	77	56	79	0.3164	212	0.3207	71	56	85	0.3225	212	0.3245
Com3	64	85	70	0.3388	219	0.3313	64	82	75	0.3396	221	0.3386

by the geometric mean.

$B=w_1^2/w_2w_3^4$ and $C=w_2^2w_3^2/w_1$ are derived from equation $[A^2B^2C, BC^2, A]^T=(w_1^3, w_2^3, w_3^3)^T$, and these two products are shown by $BC=w_1w_2/w_3^2$.

On the other hand, because the component of are eigenvector $\boldsymbol{UW}$ $z_1=\dfrac{B\{(B+AC)(1-k^2)-AC\}}{(B+AC)(1-k^2)-B}z_3$ and $z_2=\dfrac{C\{(B+AC)(1-k^2)-B\}}{(B+AC)(1-k^2)-AC}z_3$, the product of eigenvector z_1 and z_2 is taken when BC is deleted.

$$\frac{w_1}{z_1}\frac{w_2}{z_1}=\left(\frac{w_3}{z_3}\right)^2 \tag{11}$$

Other words, the consistency with the ideal solution (eigenvectors) in the fallacy of composition problem and approximating solution (geometric mean) will hint on the validity of this method, though the equation (1) is a simple form.

6. Conclusion

This paper presents a new method of dissolution of the dilemma problem by using ANP. The legitimate of this method is proven by the solution in AHP. This method is more descriptive and excellent than the method of Triantaphyllou, because the dissolution of the dilemma problem is described by ANP that clarifies the interaction of alternatives and criteria. Normalization is often available in ANP. However, the illustration of Triantaphyllou suggests having not to normalize the criteria matrix easily. It is shown that the fallacy of composition can be derived from a simple dilemma, and that the dilemma problem accompanied fallacy of composition can be dissolved by this method. This method seems to be useful to make a priority of the alternative matrix with missing values.

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Appendices

Appendix 1. Concurrent Convergence Method

The evaluation value of alternatives (K, H, N) i is assumed to be equation (A1).

$$\boldsymbol{M}=\begin{bmatrix} M_1^{K} & M_2^{HK} & M_3^{HK} \\ M_1^{KH} & M_2^{H} & M_3^{NH} \\ M_1^{KN} & M_2^{HN} & M_3^{N} \end{bmatrix} \tag{A1}$$

The ratio of the evaluation value of alternative Y and alternative X in criterion i is defined by the equation (A2).

$$M_i^{HK}=\frac{M_i^H}{M_i^K} \tag{A2}$$

$[1 \quad M_1^{KH} \quad M_1^{KN}]^T$	$[M_2^{KH} \quad 1 \quad M_2^{KN}]^T$	$[M_3^{KH} \quad M_3^{KN} \quad 1]^T$	(1)
	$[M_1^{KH} \quad 1 \quad M_1^{KN}]^T$	$[M_1^{KH} \quad M_1^{KN} \quad 1]^T$	(2)
$[1 \quad M_2^{KH} \quad M_2^{KN}]^T$		$[M_2^{KH} \quad M_2^{KN} \quad 1]^T$	(3)
$[1 \quad M_3^{KH} \quad M_3^{KN}]^T$	$[M_3^{KH} \quad 1 \quad M_3^{KN}]^T$		(4)
$\begin{bmatrix} 1 \\ \sqrt[3]{M_1^{KH} M_2^{KH} M_3^{KH}} \\ \sqrt[3]{M_1^{KH} M_2^{KH} M_3^{KH}} \end{bmatrix}$	$\begin{bmatrix} \sqrt[3]{M_1^{KH} M_2^{KH} M_3^{KH}} \\ 1 \\ \sqrt[3]{M_1^{KH} M_2^{KH} M_3^{KH}} \end{bmatrix}$	$\begin{bmatrix} \sqrt[3]{M_1^{KH} M_2^{KH} M_3^{KH}} \\ \sqrt[3]{M_1^{KH} M_2^{KH} M_3^{KH}} \\ 1 \end{bmatrix}$	(5)

The numerical value of (5) and (1) is replaced until converging.

Appendix 2. Calculating the eigenvector in AHP

Let's consider the following matrix of the dilemma problem.

$$\boldsymbol{U}=\begin{array}{r} \\ Kodama \\ Hikari \\ Nozomi \end{array}\begin{array}{c} \begin{array}{ccc} T_1 & T_2 & T_3 \end{array} \\ \begin{bmatrix} a_1 & b_1 & 0 \\ a_2 & 0 & c_1 \\ 0 & b_2 & c_2 \end{bmatrix} \end{array} \tag{A3}$$

The equation (A3) is rewritten to the form of the pair wise comparison, and the matrix $\boldsymbol{U}_{AHP}$ is represented by A, B, and C.

$$\boldsymbol{U}_{AHP} = \begin{matrix} K \\ H \\ N \end{matrix} \begin{bmatrix} a_1/a_1 & a_1/a_2 & b_1/b_2 \\ a_2/a_1 & a_2/a_2 & c_1/c_2 \\ b_2/a_1 & c_2/c_1 & c_2/c_2 \end{bmatrix} = \begin{bmatrix} 1 & A & B \\ 1/A & 1 & C \\ 1/B & 1/C & 1 \end{bmatrix}$$

Let's assume the maximum eigenvalue and the eigenvector to be α and $\boldsymbol{y}$ (y_1, y_2, y_3).

$$y_1 + Ay_2 + By_3 = \alpha y_1 \tag{A4}$$

$$y_1/A + y_2 + Cy_3 \frac{K}{H} \alpha y_2 \tag{A5}$$

$$y_1/B + y_2/C + y_{N3} = \alpha y_3 \tag{A6}$$

When the equation (A4) - the equation (A6) are arranged, the following equations are obtained.

$$(1-\alpha)y_1 + Ay_2 + By_3 = 0 \tag{A4'}$$

$$y_1/A + (1-\alpha)y_2 + Cy_3 = 0 \tag{A5'}$$

$$y_1/B + y_2/C + (1-\alpha)y_3 = 0 \tag{A6'}$$

The equation (A7) is obtained from the equation (A4') and the equation (A6').

$$y_1 = \frac{B\{AC(1-\alpha) - B\}}{B(1-\alpha) - AC} y_3 \tag{A7}$$

The equation (A8) is obtained from the equation (A5') and the equation (A6').

$$y_2 = \frac{C\{B(1-\alpha) - AC\}}{AC(1-\alpha) - B} y_3 \tag{A8}$$

By assuming $y_3 = 1$,

$$y_1 = \frac{B\{AC(1-\alpha) - B\}}{B(1-\alpha) - AC} \text{ and } y_2 = \frac{C\{B(1-\alpha) - AC\}}{AC(1-\alpha) - B} \tag{A9}$$

are obtained as the eigenvector of $\boldsymbol{U}_{AHP}$.

Appendix 3. Calculating the eigenvector in ANP

The matrix inverse of the equation (A3) is shown by the following equation.

$$\boldsymbol{U}^{-1}=\begin{bmatrix} \frac{b_2c_1}{a_1b_2c_1+a_2b_1c_2} & \frac{b_1c_2}{a_1b_2c_1+a_2b_1c_2} & \frac{-b_1c_1}{a_1b_2c_1+a_2b_1c_2} \\ \frac{a_2c_2}{a_1b_2c_1+a_2b_1c_2} & \frac{-a_2c_2}{a_1b_2c_1+a_2b_1c_2} & \frac{a_1c_1}{a_1b_2c_1+a_2b_1c_2} \\ \frac{-a_2b_2}{a_1b_2c_1+a_2b_1c_2} & \frac{a_1b_2}{a_1b_2c_1+a_2b_1c_2} & \frac{a_2b_1}{a_1b_2c_1+a_2b_1c_2} \end{bmatrix} \tag{A10}$$

Let's think about the evaluation matrix $\boldsymbol{W}$ from whom element of (1, 3), (2, 2), and (3, 1) of the matrix inverse $\boldsymbol{U}^{-1}$ is replaced by zero.

$$\boldsymbol{W}=\begin{bmatrix} \frac{b_2c_1}{a_1b_2c_1+a_2b_1c_2} & \frac{b_1c_2}{a_1b_2c_1+a_2b_1c_2} & 0 \\ \frac{a_2c_2}{a_1b_2c_1+a_2b_1c_2} & 0 & \frac{a_1c_1}{a_1b_2c_1+a_2b_1c_2} \\ 0 & \frac{a_1b_2}{a_1b_2c_1+a_2b_1c_2} & \frac{a_2b_1}{a_1b_2c_1+a_2b_1c_2} \end{bmatrix} \tag{A11}$$

Calculate the maximum eigenvalue and eigenvector $\boldsymbol{x}$ and $\boldsymbol{z}$ in ANP.

$$\begin{bmatrix} \boldsymbol{0} & \boldsymbol{W} \\ \boldsymbol{U} & \boldsymbol{0} \end{bmatrix}\begin{bmatrix} \boldsymbol{x} \\ \boldsymbol{z} \end{bmatrix}=k\begin{bmatrix} \boldsymbol{x} \\ \boldsymbol{z} \end{bmatrix} \tag{A12}$$

From $\boldsymbol{Wz}=k\boldsymbol{x}$ and $\boldsymbol{Ux}=k\boldsymbol{z}$, $\boldsymbol{UWz}=k^2\boldsymbol{z}$ is as follows.

$$\boldsymbol{UWz}=\begin{bmatrix} a_1 & b_1 & 0 \\ a_2 & 0 & c_1 \\ 0 & b_2 & c_2 \end{bmatrix}\begin{bmatrix} \frac{b_2c_1}{a_1b_2c_1+a_2b_1c_2} & \frac{b_1c_2}{a_1b_2c_1+a_2b_1c_2} & 0 \\ \frac{a_2c_2}{a_1b_2c_1+a_2b_1c_2} & 0 & \frac{a_1c_1}{a_1b_2c_1+a_2b_1c_2} \\ 0 & \frac{a_1b_2}{a_1b_2c_1+a_2b_1c_2} & \frac{a_2b_1}{a_1b_2c_1+a_2b_1c_2} \end{bmatrix}\cdot\begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix}$$

$$=\begin{bmatrix} 1 & \frac{a_1b_1c_2}{a_1b_2c_1+a_2b_1c_2} & \frac{a_1b_1c_1}{a_1b_2c_1+a_2b_1c_2} \\ \frac{a_2b_2c_1}{a_1b_2c_1+a_2b_1c_2} & 1 & \frac{a_2b_1c_1}{a_1b_2c_1+a_2b_1c_2} \\ \frac{a_2b_2c_2}{a_1b_2c_1+a_2b_1c_2} & \frac{a_1b_2c_2}{a_1b_2c_1+a_2b_1c_2} & 1 \end{bmatrix}\begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} \tag{A13}$$

Each element is calculated as follows;

$$(1-k^2)z_1+\frac{a_1b_1c_2}{a_1b_2c_1+a_2b_1c_2}z_2+\frac{a_1b_1c_1}{a_1b_2c_1+a_2b_1c_2}z_3=0 \tag{A14}$$

$$\frac{a_2b_2c_1}{a_1b_2c_1+a_2b_1c_2}z_1+(1-k^2)z_2+\frac{a_2b_1c_1}{a_1b_2c_1+a_2b_1c_2}z_3=0 \tag{A15}$$

$$\frac{a_2b_2c_2}{a_1b_2c_1+a_2b_1c_2}z_1+\frac{a_1b_2c_2}{a_1b_2c_1+a_2b_1c_2}z_2+(1-k^2)z_3=0 \tag{A16}$$

From the equation (A14) and the equation (A16),

$$z_1=\frac{\frac{b_1}{b_2}\left\{(1-k^2)-\frac{a_1 b_2 c_1}{a_1 b_2 c_1+a_2 b_1 c_2}\right\}}{(1-k^2)-\frac{a_2 b_1 c_2}{a_1 b_2 c_1+a_2 b_1 c_2}}z_3 \tag{A17}$$

is obtained.

From the equation (A14) and the equation (A15),

$$z_2=\frac{\frac{c_1}{c_2}\left\{(1-k^2)-\frac{a_2 b_1 c_2}{a_1 b_2 c_1+a_2 b_1 c_2}\right\}}{(1-k^2)-\frac{a_1 b_2 c_1}{a_1 b_2 c_1+a_2 b_1 c_2}}z_3 \tag{A18}$$

is obtained.

When the quations (A17) and (A18) are rewriten by A, B, C and $1-\frac{1}{1-k^2}=\beta-1$ under the $z_3=1$,

$$z_1=\frac{B\{(B+AC)(1-k^2)-AC\}}{(B+AC)(1-k^2)-B}=\frac{B\{B-(1-\beta)AC\}}{AC-(1-\beta)B}=\frac{B\{(1-\beta)AC-B\}}{(1-\beta)B-AC}=y_1$$

$$z_2=\frac{C\{(B+AC)(1-k^2)-B\}}{(B+AC)(1-k^2)-AC}=\frac{C\{AC-(1-\beta)B\}}{B-(1-\beta)AC}=\frac{C\{(1-\beta)B-AC\}}{(1-\beta)AC-B}=y_2$$

are obtained. Therefore, the eigenvector of the equation (A13) becomes equivalent with the eigenvector of $\boldsymbol{U}_{AHP}$.